# On Two Different Kinds of Computational Indeterminacy 



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## Radical Indeterminacy (triviality results)

- "Theorem: Every ordinary open system [that satisfies two minimal principles] is a realization [implementation] of every abstract finite automaton" (Putnam 1988: 121).
- "For any program and for any sufficiently complex object, there is some description of the object under which it is implementing the program" (John Searle 1992: 208).

Pancomputationalism: Every physical system performs every computation.

## Avoiding triviality (but not indeterminacy)

- Most critics: Putnam and Searle assume an overly liberal notion of implementation, arguing that it takes more than simple homomorphism to implement an automaton.


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- But, computational indeterminacy is here to stay: Even with the additional constraints, some physical systems perform, simultaneously, more than one computation.
- The phenomenon is well-known, and described by various names ('simultaneous implementation', 'the ambiguity of representation', 'indeterminacy of computation', 'underdetermination of computation', 'multiple-computations theorem', 'multiplicity of computations'...)


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- Most critics: Putnam and Searle assume an overly liberal notion of implementation, arguing that it takes more than simple homomorphism to implement an automaton.
- But, computational indeterminacy is here to stay: Even with the additional constraints, some physical systems perform, simultaneously, more than one computation.
- The phenomenon is well-known, and described by various names ('simultaneous implementation', 'the ambiguity of representation', 'indeterminacy of computation', 'underdetermination of computation', 'multiple-computations theorem', 'multiplicity of computations'...)
- But it's traditionally been regarded as a unified phenomenon. We argue that it's not.


# The proposal: <br> Two kinds of computational indeterminacy 

Functional indeterminacy<br>Interpretative indeterminacy

## Functional indeterminacy

Functional indeterminacy concerns a functional (or formal) characterization of the system's relevant behavior (briefly: how its physical states are grouped together and corresponded to abstract states).

The indeterminacy claim is that there are several ways of grouping physical states together, such that each particular way can provide the computational structure (or "vehicle") of the physical system.

## Interpretative indeterminacy

Interpretative indeterminacy concerns the manner in which the abstract states are interpreted (briefly: what is the formal, mathematical or logical, content of these abstract states).

The indeterminacy claim is that there are several ways of assigning formal content to the abstract/functional states of the system; under each assignment (interpretation) the system computes a different mathematical or logical function.

## An example: A tri-stable system $P$

Input 1

Input 2


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |

## How to computationally characterize $P$ ?

Input 1

Input 2


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |

## The functional organization of $P$



| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| A | C | A |
| B | A | A |
| B | B | B |
| B | C | B |
| C | A | A |
| C | B | B |
| C | C | C |

## The $\boldsymbol{A}$ functional organization of $P$



| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| A | C | A |
| B | A | A |
| B | B | B |
| B | C | B |
| C | A | A |
| C | B | B |
| C | C | C |

## Additional possible groupings (coarser-grained)

But, there is no inherent reason to assume that the previous grouping is the only one. Consider:

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |

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| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |

$$
\begin{gathered}
A=\{7-10 \mathrm{~V}\} \\
B=\{0-3 \mathrm{~V} ; 4-6 \mathrm{~V}\} \\
\quad---7
\end{gathered}
$$

## Additional possible grouping (coarser-grained)

But, there is no inherent reason to assume that the previous grouping is the only one. Consider:

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |

$$
\begin{aligned}
A & =\{7-10 \mathrm{~V}\} \\
B & =\{0-3 \mathrm{~V} ; 4-6 \mathrm{~V}\} \\
& =--7
\end{aligned}
$$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |

## Additional possible grouping (coarser-grained)

Also consider:

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |

## Additional possible grouping (coarser-grained)

Also consider:

| Input 1 | Input 2 | Output | $\begin{aligned} & A^{\prime}=\{4-6 \mathrm{~V} ; 7-10 \mathrm{~V}\} \\ & \mathrm{B}^{\prime}=\{0-3 \mathrm{~V}\} \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | 4-6 V |  |
| $7-10 \mathrm{~V}$ | 4-6 V | 7-10 V |  |
| $7-10 \mathrm{~V}$ | 0-3 V | $7-10 \mathrm{~V}$ |  |
| 4-6 V | $7-10 \mathrm{~V}$ | 7-10 V |  |
| $4-6 \mathrm{~V}$ | 4-6 V | $4-6 \mathrm{~V}$ | $\cdots$ |
| $4-6 \mathrm{~V}$ | 0-3 V | $4-6 \mathrm{~V}$ |  |
| 0-3 V | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |  |
| 0-3 V | 4-6 V | $4-6 \mathrm{~V}$ |  |
| 0-3 V | 0-3 V | 0-3 V |  |

## Additional possible grouping (coarser-grained)

Also consider:

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |


| $\begin{aligned} & A^{\prime}=\{4-6 \mathrm{~V} ; 7-10 \mathrm{~V}\} \\ & \mathrm{B}^{\prime}=\{0-3 \mathrm{~V}\} \end{aligned}$ | Input 1 | Input 2 | Output |
| :---: | :---: | :---: | :---: |
|  | A' | A' | A' |
|  | A' | B' | $A^{\prime}$ |
| ---7 | B' | A' | A' |
|  | B' | B' | B' |

## Functional indeterminacy

- The physical system $P$ can be seen as having (at least) three different functional profiles (or as implementing three different automata).
- The indeterminacy claim is that each such profile can be seen (at least potentially) as the computational structure (or "vehicle") of the system.
- The philosophical debate is on whether all these functional profiles are also actual computational profiles, and, if not, what is the ingredient that singles out the computational structure.


## Interpreting the functional states

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |

## Interpreting the functional states

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |

$\underset{B^{\prime} \rightarrow \mathrm{F}}{\mathrm{A}^{\prime} \rightarrow \mathrm{T}} \quad \swarrow$


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## Interpreting the functional states

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |



】 $\begin{aligned} & A^{\prime} \rightarrow F \\ & B^{\prime} \rightarrow T\end{aligned}$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| F | F | F |
| F | T | F |
| T | F | F |
| T | T | T |

## Interpreting the functional states

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |


】 $\quad \begin{aligned} & A^{\prime} \rightarrow F \\ & B^{\prime} \rightarrow T\end{aligned}$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| F | F | F |
| F | T | F |
| T | F | F |
| T | T | T |

## Interpretative indeterminacy

- The OR/AND difference surfaces when we interpret the same abstract/functional structure in different ways.
- The indeterminacy claim is that the system can be seen as computing OR and as computing AND.
- Fresco, Copeland and Wolf (2021) is about re-interpreting the same functional states, which results in "dual functions", where you switch the truth-values.


## More on interpretative indeterminacy

- In the examples of interpretative indeterminacy that are found in the literature the range of the interpretations is limited to universes that are populated by formal, mathematical, logical or other abstract entities, such as numbers, sets, or truth-values.
- The philosophical debate is on whether interpretative indeterminacy reflects computational indeterminacy, and, if yes, what is the ingredient that singles out the computational structure.


# Clarifying functional vs. interpretative indeterminacy 

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |

## Clarifying functional vs. interpretative indeterminacy

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |

$A=\{7-10 \mathrm{~V}\}$
$B=\{0-3 \mathrm{~V} ; 4-6 \mathrm{~V}\}$
---7

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |

Bob:

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $7-10 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $4-6 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $7-10 \mathrm{~V}$ | $7-10 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $4-6 \mathrm{~V}$ | $4-6 \mathrm{~V}$ |
| $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ | $0-3 \mathrm{~V}$ |


|  | Anput 1 | Input 2 | Output |
| :--- | :---: | :---: | :---: |
| $\mathrm{A}^{\prime}=\{4-6 \mathrm{~V} ; 7-10 \mathrm{~V}\}$ |  |  |  |
| $\mathrm{B}^{\prime}=\{0-3 \mathrm{~V}\}$ |  |  |  |
|  | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
|  | $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
|  | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
|  | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |

## Clarifying functional vs. interpretative indeterminacy

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |

## Clarifying functional vs. interpretative indeterminacy

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~T} \\
& \mathrm{~B} \rightarrow \mathrm{~F}
\end{aligned}
$$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

## Clarifying functional vs. interpretative indeterminacy

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~T} \\
& \mathrm{~B} \rightarrow \mathrm{~F}
\end{aligned}
$$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

## Clarifying functional vs. interpretative indeterminacy

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~T} \\
& \mathrm{~B} \rightarrow \mathrm{~F}
\end{aligned} \downarrow
$$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |

$$
\downarrow \begin{aligned}
& \mathrm{A}^{\prime} \rightarrow \mathrm{T} \\
& \mathrm{~B}^{\prime} \rightarrow \mathrm{F}
\end{aligned}
$$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

## Clarifying functional vs. interpretative indeterminacy

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| A | A | B |
| A | B | A |
| B | A | A |
| B | B | B |

$$
\begin{aligned}
& \mathrm{A} \rightarrow \mathrm{~T} \\
& \mathrm{~B} \rightarrow \mathrm{~F}
\end{aligned} \downarrow
$$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |


| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{A}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{A}^{\prime}$ | $\mathrm{A}^{\prime}$ |
| $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ | $\mathrm{B}^{\prime}$ |

$$
\downarrow \begin{aligned}
& \mathrm{A}^{\prime} \rightarrow \mathrm{T} \\
& \mathrm{~B}^{\prime} \rightarrow \mathrm{F}
\end{aligned}
$$

| Input 1 | Input 2 | Output |
| :---: | :---: | :---: |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

So $P$ can be seen as computing XOR and OR. Functional or interpretative indeterminacy?

## Radical computational indeterminacies

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## Radical computational indeterminacies

- When there are (almost) no constraints on the groupings of physical states, the functional indeterminacy deteriorates to triviality results (every physical system implements every automaton).
- When there are (almost) no constraints on the interpretations of abstract states, the interpretative indeterminacy deteriorates to "deviant encoding", in which every physical system computes every number-theoretic function.


## Summary

- We characterized and distinguished between two kinds of computational indeterminacy, which has traditionally been regarded as a unified phenomenon.
- One kind is a functional indeterminacy, which has to do with different groupings of physical properties into different abstract structures. The other kind of indeterminacy has to do with the interpretation of the physical/abstract states of the system.


